



THEORETICAL ANALYSIS OF THE EFFECT OF THE LINTER WORKING-CHAMBER VOLUME ON ITS PRODUCTIVITY

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Abstract. This article presents a theoretical study of the complex and stochastic nature of the linting process and the motion of cotton seeds within the working chamber of a linter. The motion of seeds is analyzed in terms of closed trajectories and isolated periodic solutions, with consideration given to the application of the Poincaré–Bendixson and Dulac criteria. Different mechanisms of lint removal from the seed surface are examined, including the interaction between saw teeth and seeds, as well as the influence of seed-roll density and relative movement speeds on the linting process. The theoretical analysis demonstrates that an increase in the cross-sectional area and, consequently, the volume of the linter’s working chamber can increase the productivity of the linter in terms of lint removal and seed throughput.

Keywords: linter; linting process, cotton seed, working chamber; working-chamber volume, seed roll, saw cylinder, saw tooth, lint, short fibres, closed trajectory, periodic solution, Poincaré–Bendixson criterion, Dulac criterion, productivity, seed throughput.

INTRODUCTION

It is well known that the linting process is highly complex and stochastic in nature. The motion of the seeds within the linter’s working chamber is, in many respects, unpredictable. It can be characterised as the motion of particles inside a closed vessel.

To analyse this motion, let us consider the following situation. In essence, every fibred seed must come into contact with the saw, and the saw must scrape the fibre from it.

In the ginning process, entanglement often occurs because the fibres are long enough to stick to one another; in linting, by contrast, no such process occurs, because the seeds coming out of ginning already carry fibres of much shorter length. Here, therefore, the essential process is the scraping of the down (short fibre) from the seed surface. In this case the seed needs to move considerably less relative to the saw than it does during ginning.

The motion of the seeds within the chamber volume can be represented as an isolated closed trajectory – that is, a trajectory free of neighbouring points lying on other closed trajectories. This second circumstance is directly linked to the question of whether isolated periodic solutions exist. It is notable that isolated closed trajectories can occur only in nonlinear differential equations and systems.

MAIN PART

Isolated periodic solutions correspond to the most diverse features of the phenomena and processes that occur as the seed moves within the mass. In studying the processes that occur in the seed roll, we examine the possibility of isolated periodic solutions; here, as a model, we consider the following nonlinear differential system:

$$\begin{cases} \frac{dx}{dt} = -y + x(1 - x^2 - y^2) \\ \frac{dy}{dt} = x + y(1 - x^2 - y^2) \end{cases} \quad (1)$$

To solve it, we introduce the polar coordinates r and θ , where $x = r \cdot \cos \theta$, $y = r \cdot \sin \theta$; then, by differentiating the relations $r^2 = x^2 + y^2$ and $\theta = \arctan(y/x)$ with respect to t , we obtain the following relations (Chumakov, 2007):

$$\begin{cases} x \cdot \frac{dx}{dt} + y \cdot \frac{dy}{dt} = r \cdot \frac{dr}{dt} \\ x \cdot \frac{dy}{dt} - y \cdot \frac{dx}{dt} = r^2 \cdot \frac{d\theta}{dt} \end{cases} \quad (2)$$

Multiplying the first equation of system (1) by x and the second by y , and adding them together taking into account the first relation from (2), we obtain the following equality:

$$r \cdot \frac{dr}{dt} = r^2(1 - r^2) \quad (3)$$

If we multiply the second equation of system (2) by x and the first by y , and subtract them, taking into account the second relation from (3), we obtain the following ratio:

$$r \cdot \frac{d\theta}{dt} = r^2 \quad (4)$$

System (1) has a single equilibrium point $O(0, 0)$. Since we are interested only in constructing trajectories, we may assume that $\rho > 0$. Then, taking equations (3) and (4) into account, system (1) reduces to the following form:

$$r \cdot \frac{dr}{dt} = r^2(1 - r^2), \quad r \cdot \frac{d\theta}{dt} = 1 \quad (5)$$

Each of the resulting equations of system (5) is readily integrated, and the entire family of solutions is given by the formula:

$$r = \frac{1}{\sqrt{1 + Ce^{-2t}}}, \quad \theta = t + t_0 \quad (6)$$

Or the variables x and y can be presented in the following form:

$$x = \frac{A \cos(t + t_0)}{\sqrt{1 + Ce^{-2t}}}, \quad y = \frac{A \sin(t + t_0)}{\sqrt{1 + Ce^{-2t}}} \quad (7)$$

Suppose that, in the motion, we have a bounded region d that, together with its boundary, lies in the phase plane and contains none of the equilibrium points of system (2). Then, by the Poincaré–Bendixson criterion, if a trajectory r of system (2) leaves, at the initial instant $t = t_0$, a point lying in the region and remains in d for all $t > t_0$, then the trajectory r is itself a closed trajectory, or, with time, it approaches a closed trajectory in a spiral (Amel'kin, 1987; Katok & Hasselblatt, 1999).

Let us plot the motion graphs of the seed roll shown in Figure 1. Here d consists of two closed curves, r_1 and r_2 , and the annular region between them.

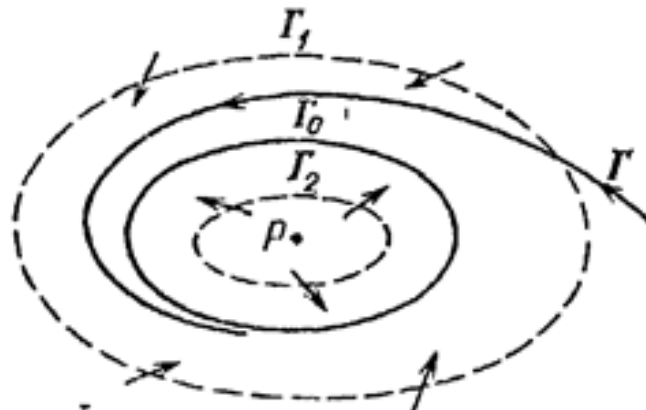


Figure 1. Diagram of the closed-contour motion of the seed in the linter chamber.

We associate a vector with each boundary point of the domain d :

$$V(x, y) = X(x, y) \cdot i + Y(x, y) \cdot j \quad (8)$$

If the trajectory r leaves a boundary point at the initial instant $t = t_0$ and enters the region d , and remains there entirely for $t > t_0$, then, according to the conclusion given above, the trajectory r approaches some closed trajectory r_0 lying in the region d . In this case the curve r_0 encloses a singular point (point P) of the differential system that does not lie in the region d .

Differential system (2) gives a simple example of applying the above criterion to find limit cycles. Indeed, the system has a single equilibrium point $O(0, 0)$, and therefore, in the region d between the circles of radius $r = 1/2$ and $r = 2$, there are no equilibrium points. Based on the first equation of system (4), we find that $dr/dt > 0$ on the inner circle and $dr/dt < 0$ on the outer circle. The vector V associated with the points on the boundary of region d is always directed into the interior of d . This means that, in the annular region between the circles of radius $r = 1/2$ and $r = 2$, differential system (2) must have a closed trajectory. Such a closed trajectory does indeed exist, and it is the circle of radius $r = 2$.

It should be noted, however, that in the general case of a system of form (2) the practical application of the Poincaré–Bendixson criterion runs into considerable difficulty, since there is no general method for constructing the required domains, and success therefore depends both on the type of the system and on the experience of the researcher. At the same time, it should be kept in mind that finding evidence of the absence of limit cycles is a task no less important than finding criteria for their existence.

The most widely used criterion in this regard is Dulac's criterion: if there exists a continuously differentiable function $B(x, y)$ such that, in some simply connected domain d of the phase plane, the function $d(BX)/dx + d(BY)/dy$ is of constant sign, then differential system (2) has no limit cycles in the domain d . When $B(x, y) = 1$, this reduces to Bendixson's criterion (Amel'kin, 1987).

If we picture the motion of the seed, the equations of motion can be expressed by a stochastic function describing trajectories that spiral inward, all of which depend on the trajectory of motion. Moving along the trajectory, the seeds move unevenly, because this is linked to the density of the seed roll. They may lag behind and then stop, which causes the seed to slip on the saw, and this results in the down being scraped off the seed.

In some cases these stoppages are related to the huller-rib grid; in other cases, as mentioned above, they are simply a manifestation of the uneven motion of the mass holding the seed against the saw. Let us first consider the case in which the down is cut from the seed. To do this, let us picture the saw tooth as a blade and convert the diagram into the correct form.

The cutting process can be observed to occur in different ways: in the first case the fibre itself is separated from the seed; in the second, the seed's surface is scraped by the saw tooth and the seed coat is damaged; and in the third case the fibres are cut cleanly. Let us consider each of these separately. In the first case the fibre's contact with the saw occurs somewhat away from the seed, where it adheres to a cluster of neighbouring seeds (Figure 2).

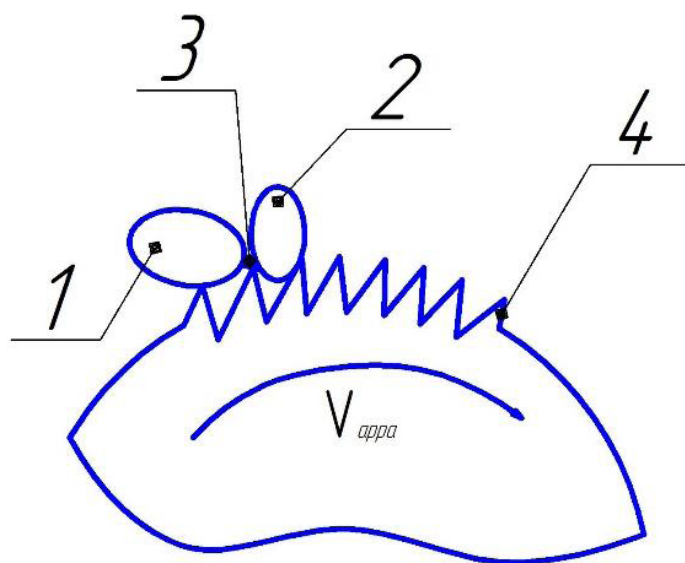


Figure 2. Diagram of the process of scraping the down from the seed.

In this case separation of the down occurs when the seed roll, under compression, presses the fibre against the seed. Given that the seed has a cross-section equal to d , and that the seed-roll load on an element of area equal to ds presses upon it, the fibre-holding force T , with which the fibre is separated, can be calculated – with μ as the coefficient – using the following formula:

$$T = F_{fr} = \mu \cdot d \cdot ds \cdot g \quad (9)$$

On the basis of expression (9) it can be said that the larger the area of the chamber, the greater the fibre-holding force can be.

In the second case the down is separated together with part of the seed coat. In this case the down turns out to be almost cleanly cut, but it should be noted that the area over which the fibre is removed from the surface is larger.

It can be noted that when the seed is struck in this way, more down is removed, and the seed also undergoes rotation. Suppose we have a seed of diameter d_1 and length L , with tooth height t_z and area l_1 . In such contact the amount of down scraped from the seed can be determined if we know the number of fibres I corresponding to a unit seed area of 1 mm^2 .

At the same time we see that the area of down removed from the seed surface is considerably larger and depends on the difference between the speeds of the seed and the saw. Suppose the seed moves at speed V_{seed} and the saw at speed V_{saw} , the working-zone length is L_{zone} , and we assume the most favourable case, in which the seed passes along the entire length of the working zone. Let us determine how many saw teeth pass the seed, and what the total length of the path cut along the seed will be:

$$Z = \frac{L_{\text{way}} \cdot \pi \cdot n}{V_{\text{seed}} \cdot 30 \cdot t_z} \quad (10)$$

During this time the total length of the path over which the down is scraped is equal to

$$L_{\text{way}} = Z \cdot t_z \quad (11)$$

Taking this into account, with the seed's dimension along the perimeter L_{along} and the saw width equal to h , the total maximum area of down cut is equal to:

$$S_{\text{max}} = \frac{L_{\text{cut}}}{L_{\text{along}}} \cdot h \quad (12)$$

Considering that $L_{\text{cut}} / L_{\text{along}} \geq 1$, and that the seed has freedom to rotate about its own axis, while at the same time the seed rotates at speed S_z and the seeds may push against one another – knowing the total area of the seed, we express it as the area of an equivalent sphere:

$$S_{\text{sphere}} = 4\pi R^2 \quad (13)$$

The stability coefficient of the process can be expressed as follows:

$$\frac{S_{\text{max}}}{S_{\text{sphere}}} = k_1 \quad (14)$$

For example, when $k_1 \geq 1$ the process is considered ideal; the greater k_1 is above 1, the better. In practice, however, such a process does not occur, because, as noted above, seed linting is a stochastic process – something explained, in production, by the multiple

stages of linting. Figure 3 presents the graph of the effect of the linter working-chamber volume on productivity in down removal.

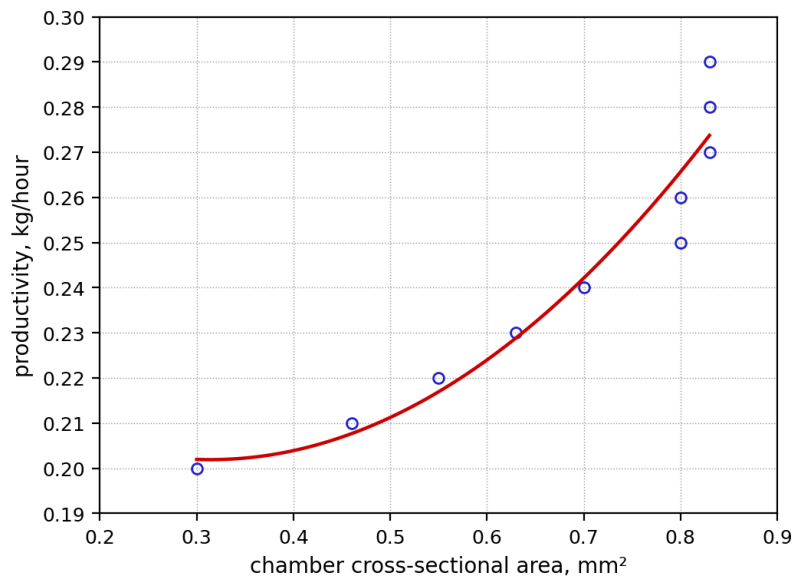


Figure 3. Graph of the effect of the linter working-chamber volume on productivity in down removal.

The graph in Figure 3 shows that, as the cross-sectional area (and, correspondingly, the volume) of the working chamber increases, the linter's productivity in down removal also increases.

In this case the total amount of down scraped from the seed in a single pass of the saw is equal to

$$m_{\text{fibre}} = \rho \cdot l \cdot S_{\text{max}} \quad (15)$$

In addition, damage to the seed may also occur in this region, depending on the magnitude of the weight pressure on the seed, since part of the mass of the seed roll presses on the seed being linted. This pressure must be less than the strength of the seed coat, which can be calculated using the following formula:

$$\sigma > \frac{P}{S_{\text{tooth}}} = \frac{H_1 \cdot dS \cdot D_{\text{seed}} \cdot \gamma \cdot g}{S_{\text{tooth}}} \quad (16)$$

where:

H_1 – height of the chamber sector, [m];

dS – area of the seed-roll mass loading over the seed, [m²];

γ – density of the seed roll;

σ – allowable strength of the seed coat, [Pa];

S_{tooth} – contact area with the saw, [m²].

$$\tau = \frac{P_{\text{tens}}}{S_{\text{csa}}} \quad (17)$$

where:

P_{tens} – tensile force of the fibre;

S_{csa} – cross-sectional area, [mm²].

CONCLUSION

As a result of the theoretical study of the effect of the working-chamber volume of the improved saw-type linter unit on its productivity – in terms of down removal and seed throughput during the seed-linting process – equations were derived that describe the closed-loop motion of the seed layer interacting with the saw tooth in the working chamber, and that describe the process of scraping the down. It was established that increasing the cross-sectional area of the linter's working chamber (that is, the working-chamber volume) makes it possible to increase the productivity of the linter unit.

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